ADIABATIC TURBULENT FLOW OF A GAS IN A CYLINDRICAL TUBE

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A solution is given for one-dimensional adiabatic turbulent flow of a compressible gas in a cylindrical tube, taking into account variation of the Re number along the tube and the influence of compressibility on the friction coefficient. The solution is compared with formulas derived in [1] and [2].

Adiabatic flow of a gas in a cylindrical tube (one-dimensional model) is described by the set of equations

$$dp + \rho w dw + \zeta \frac{\rho w^2}{2} \frac{dx}{D} = 0,$$
$$T_0 = T\left(1 - \frac{k-1}{k+1}\lambda^2\right) = \text{const}, \quad p = \rho RT, \tag{1}$$
$$G = \rho w F = \text{const},$$

from which it is easy to obtain the change of relative velocity $\lambda = \omega/\alpha_*$ with the distance along the tube x and the friction coefficient ζ [1]:

$$\frac{k+1}{k}\left(\frac{1}{\lambda^2} - 1\right)\frac{d\lambda}{\lambda} = \zeta d\bar{x}. \tag{2}$$

In regions of subsonic gas flow the influence of compressibility of the gas on the friction coefficient, and the change of Re number along the tube length can be neglected in the solution of (2). At supersonic gas velocities both these factors have an appreciable influence on the nature of the gas flow along the tube.

Reference [2] gives a solution of (2) for zero heat transfer across the walls, taking into account the influence of the Mach number on the friction coefficient in accordance with [3]. Allowance for the influence of the Mach number M on the coefficient ζ in [3] was made in terms of local values of Re. In [2], however, variation of Re along the tube was neglected.

It is easy to show that when the Mach number varies from 5 to 1, the Re number calculated from the thermodynamic temperature may increase by approximately a factor of three. Consequently the local friction coefficient also changes appreciably.

Since $\rho\omega$ = const for a channel of constant section, variation of Re number along the length of the tube occurs only as a result of change in dynamic viscosity with change in temperature. For simplicity of analysis, a power law dependence of the dynamic viscosity on temperature will be assumed:

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{0.75}. \tag{3}$$

The local friction coefficient for turbulent flow may be written, in conformity with [3], in the form

$$\zeta = \zeta_{\text{н}}\left(1 - \frac{k-1}{k+1}\lambda^2\right)^{0.5}. \tag{4}$$

where the friction coefficient for flow of an incompressible fluid in a cylindrical channel is given by

$$\zeta_{\text{н}} = 0.178\,\mathrm{Re}^{-0.2}, \tag{5}$$

which is in good agreement with Nikuradze's formula in the range Re = $5 \cdot 10^4 - 10^6$.

Since the quantity ζ varies along the tube, we shall express it in terms of the value of the friction coefficient at the beginning of the tube ζ_1, taking into account relations (3)–(5).

$$\zeta = 0.178\,\mathrm{Re}_1^{-0.2}\left(1 - \frac{k-1}{k+2}\,\lambda_1^2\right)^{-0.15}\left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^{0.65} \cong$$

$$\cong 0.178\,\mathrm{Re}_1^{-0.2}\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{-0.15}\sqrt[3]{\left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^2}. \qquad (6)$$

Substituting (6) in (2), we obtain

$$\left(\frac{1}{\lambda^2} - 1\right)\frac{d\lambda}{\lambda\sqrt[3]{\left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^2}} = A\,dx, \qquad (7)$$

where

$$A = 0.178\,\frac{k}{k+1}\,\mathrm{Re}_1^{-0.2}\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{-0.15}.$$

The integral of (7) for a given Re_1 value has the form

$$\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{0.15}\left\{\frac{k+1}{2k}\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3}\lambda_1^{-2} - \left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^{1/3}\lambda^{-2}\right] - \right.$$

$$- \frac{k+5}{4k}\,\ln\left[\left(1 + \frac{k-1}{k+1}\,\lambda^2\right)^{1/3} - 1\right](\lambda_1^2)^{1/3}\left\{\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3} - 1\right]\times\right.$$

$$\times(\lambda^2)^{1/3}\Big\}^{-1} - \frac{k+5}{2\sqrt{3}\,k}\left[\mathrm{arctg}\,\sqrt{3}\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3}\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3} + 2\right]^{-1} + \right.$$

$$\left.\left. + \mathrm{arctg}\,\sqrt{3}\left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^{1/3}\left[\left(1 - \frac{k-1}{k+1}\,\lambda^2\right)^{1/3} + 2\right]^{-1}\right]\right\} = \zeta_{H1}\,\bar{x}. \qquad (8)$$

Fig. 1 gives the results of calculations based on (8) for k = 1.4, $\lambda_1 = 2.2$ ($M_1 = 4.56$). It is seen from the graph that neglecting the variation of Re along the tube leads to an overestimate of λ along the whole tube, the overestimate being the greater, the greater the values of λ_1 and Re_1 in the initial flow. The critical tube length $\bar{x}_{cr}$ is given by

$$\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{0.15}\left\{\frac{k+1}{2k}\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3}\lambda_1^{-2} - \left(1 - \frac{k-1}{k+1}\right)^{1/3}\right] - \right.$$

$$- \frac{k+5}{4k}\,\ln\left[\left(1 - \frac{k-1}{k+1}\right)^{1/3} - \right.$$

$$\left. - 1\right](\lambda_1^2)^{1/3}\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3} - \right.$$

$$\left. - 1\right]^{-1} - \frac{k+5}{2\sqrt{3}\,k}\left[\mathrm{arctg}\,\sqrt{3}\left(1 - \right.\right.$$

$$\left. - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3}\left[\left(1 - \frac{k-1}{k+1}\,\lambda_1^2\right)^{1/3} + \right.$$

$$\left. + 2\right]^{-1} + \mathrm{arctg}\,\sqrt{3}\left(1 - \right.$$

$$\left. - \frac{k-1}{k+1}\right)^{1/3}\left[\left(1 - \frac{k-1}{k+1}\right)^{1/3} + \right.$$

$$\left.\left.\left. + 2\right]^{-1}\right]\right\} = \zeta_{H1}\,\bar{x}_{cr}. \qquad (9)$$

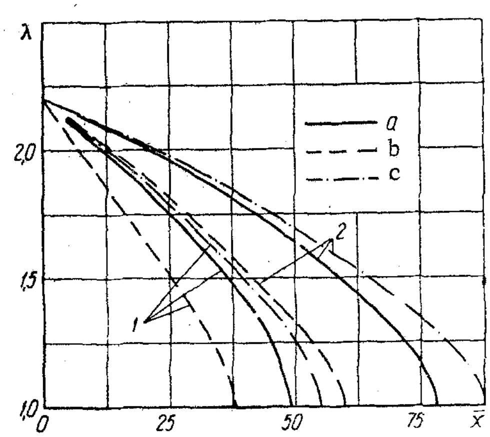

Fig. 1. Variation of λ along tube: a–according to (8); b–[1]; c–[2]; 1–for Re = 10^5; 2–10^6

Fig. 2 shows $\bar{x}_{cr}$ as a function of Re at the tube entrance. It is clear that for a given λ_1, as Re_1 increases, the length of tube with supersonic flow grows considerably.

One of the basic measured parameters in experimental investigations of the adiabatic flow of gases in tubes is the static pressure distribution along the tube. The relative pressure for an arbitrary tube section is given in [1] in the form

$$\frac{p}{p_{01}} = \frac{\lambda_1}{\lambda}\left(1 - \frac{k-1}{k+1}\lambda^2\right)\left(1 - \frac{k-1}{k+1}\lambda_1^2\right)^{\frac{1}{k-1}}, \qquad (10)$$

where p_{01} is the stagnation pressure at the tube inlet section.

Fig. 3 gives the results of calculations based on (10), taking (8) into account.

The graph shows that, for a given λ_1, the curve of variation of static pressure along the tube, calculated in accordance with (8) and (10), is steeper than that from [2], i.e., change of Re along the tube leads to a sharper rise in static pressure.

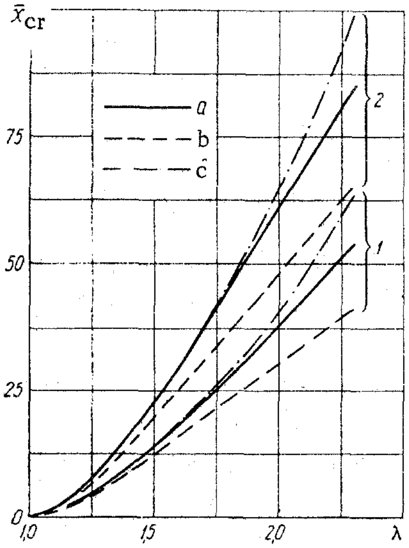

Fig. 2. Critical tube length as a function of λ: 1, 2, a, b, c – see Fig. 1.

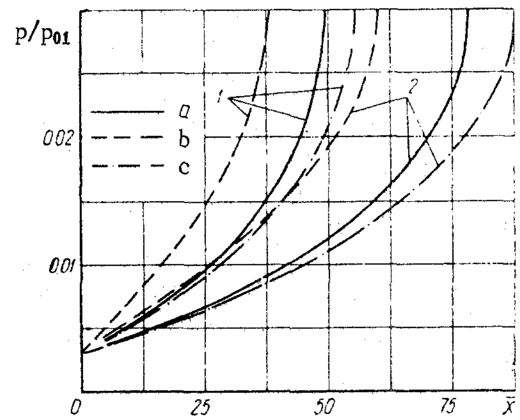

Fig. 3. Variation of pressure along tube for λ_1 = 2.2: 1, 2, a, b, c–see Fig. 1.

NOTATION

p–pressure; ρ–density; ω–velocity; a_*–critical velocity; $\lambda = \omega/a_*$–relative velocity; T–thermodynamic temperature; T_0–stagnation temperature; k–adiabatic index; G–mass flow rate; x–flow coordinate along tube axis; D–i. d. of tube; ζ–friction coefficient; μ–dynamic viscosity.

REFERENCES

1. S. A. Khristianovich, V. G. Gal'perin, M. D. Millionshchikov, and L. A. Simonov. Applied Gas Dynamics [in Russian], 1948.
2. A. I. Leont'ev and V. K. Fedorov, IFZh, No. 6, 1961.
3. F. S. Voronin, IFZh, No. 11, 1959.

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